$\boxed{10.2}$ $$\langle P \rangle = \frac{10^3 W}{4\pi (10^3 m)^2} = 7.96 \times 10^{-5} \frac{W}{m^2}$$

At this distance the radiation will appear to be a plane wave, so that

$$\vec{P} = \vec{E} \times \vec{H}$$

$$= \vec{E} \times \left(\sqrt{\frac{\epsilon_0}{\mu_0}} \hat{k} \times \vec{E} \right)$$

$$\langle P \rangle = \sqrt{\frac{\epsilon_0}{\mu_0}} \langle E^2 \rangle$$

$$= \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} E^2_{peak}$$

$$E_{peak} = \left[2 \langle P \rangle \sqrt{\frac{\mu_0}{\epsilon_0}} \right]^{1/2}$$

$$= \left[2 \times 7.96 \times 10^{-5} \frac{W}{m^2} \times \sqrt{\frac{4\pi \times 10^{-7} H.m^{-1}}{8.85 \times 10^{-12} F.m^{-1}}} \right]^{1/2}$$

$$= 0.24 \frac{V}{m}$$

$\boxed{10.3}$

$$I = \frac{V}{R_{rad} + R_{load}}$$

$$W_{load} = I^2 R_{load}$$

$$= \frac{V^2 R_{load}}{(R_{rad} + R_{load})^2}$$

$$\frac{\partial W_{load}}{\partial R_{load}} = 0 = V^2 \left[\frac{1}{(R_{rad} + R_{load})^2} - \frac{2R_{load}}{(R_{rad} + R_{load})^3} \right]$$

$$1 = \frac{2R_{load}}{R_{rad} + R_{load}}$$

$$R_{rad} = R_{load}$$

Therefore

$$W_{load} = \frac{V^2}{4R_{rad}}$$

or for a periodic signal

$$\langle W_{load} \rangle = \frac{\langle V^2 \rangle}{4R_{rad}}$$

$$= \frac{V^2_{peak}}{8R_{rad}}$$

10.4 The gain is

$$G \equiv \max_{\theta,\varphi} \frac{P(r=1,\theta,\varphi)}{W/4\pi}$$

$$= \frac{I_0^2 k^2 d^2}{32\pi^2 r^2} \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{3}{I_0^2 \pi} \sqrt{\epsilon_0 \mu_0} \left(\frac{\lambda}{d}\right)^2 4\pi$$

$$= \frac{3k^2\lambda^2}{8\pi^2}$$

$$= \frac{3}{2}$$

Since the peak voltage is related to the peak electric field by $V_{peak} = E_{peak}\, d$, the area is

$$\langle W_{load} \rangle = \frac{V_{peak}^2}{8R_{rad}}$$

$$= \frac{E_{peak}^2 d^2}{8R_{rad}}$$

$$= \frac{E_{peak}^2 d^2}{8} \frac{3}{2\pi} \sqrt{\frac{\epsilon_0}{\mu_0}} \left(\frac{\lambda}{d}\right)^2$$

$$= \langle P \rangle A$$

$$= \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} E_{peak}^2 A$$

$$\Rightarrow A = \frac{3}{8\pi} \lambda^2$$

Therefore

$$\frac{A}{G} = \frac{3}{8\pi} \lambda^2 \frac{2}{3} = \frac{\lambda^2}{4\pi}$$